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Analytical Solution of Non-Linear Equation in Multi-Phase Microchannel Bioreactors

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Abstract— *A theoretical model of multi-phase microchannel bioreactor is presented. Further, the effect of oxygen diffusion on the concentration profile and effectiveness response was examined. Analytical expression pertaining to the oxygen concentration profile and effectiveness responses was reported for all possible values of reaction diffusion parameter and the saturation parameter. These analytical results were found to be in good agreement with numerical simulations. Moreover, herein we employ new approach of homotopy perturbation method (NHPM) to solve the non-linear reaction/diffusion equation.*

Keywords— Mathematical modelling, Bioreactors, Immobilization of Gluconobacter, New homotopy perturbation method, Reaction/Diffusion equation.

I. INTRODUCTION

A wide variety of physical systems are modelled by nonlinear systems of differential equations depending upon second and, occasionally, even higher order derivatives of the unknowns. But there is an easy device that will reduce any higher order ordinary differential equation or system to an equivalent first order system. "Equivalent" means that each solution to the first order system uniquely corresponds to a solution to the higher order equation and vice versa. The upshot is that, for all practical purposes, one only needs to analyse first order systems. Moreover, the vast majority of numerical solution algorithms are designed for first order systems, and so to numerically integrate a higher order equation, one must place it into an equivalent first order form. Non-linear differential equations have long been an important area of study because of their wide application in physics, engineering, biology, chemistry, ecology, and economics. [1].

Sevukaperumal and Rajendran have derived the analytical expression of concentration and effectiveness factor of the reactant inside the catalyst pellets by solving non-linear equation using the Adomian decomposition method[2]. Saravana Kumar et al. [3], have obtained the analytical expression for the steady state concentration of substrate in terms of the reaction/diffusion parameter, in a packed bed reactor using modified adomian decomposition method. Sevukaperumal et. al [4] have solved the time independent non-linear reaction/diffusion equation in immobilized enzyme system and obtain an approximate analytical expression for the concentration and effectiveness factor under steady state conditions using Homotopy perturbation method. Thiagarajan et al. [5], have presented the approximate expression of mediated concentration using Homotopy perturbation method.

However, to the best of our knowledge, there were no simple analytical expression for the oxygen concentration for all possible values of diffusion parameter ϕ_{cc}^2 and the saturation parameter β. Therefore, herein, we employ new homotopy perturbation method to evaluate the steady-state substrate concentration and effectiveness factor for all values of diffusion and saturation parameters. we compared this analytical result with the numerical result and previous analytical result of oxygen concentration obtained by Adomian decomposition method [6] .

II. MATHEMATICAL FORMULATION OF THE PROBLEM

A general scheme that represents the reaction occurring at a multi-phase microchannel bioreactor can be represented as follows [6]:

$$\text{D}-\text{sorbitol}+(1/2))\,O_2 \rightarrow \text{L}-\text{sorbose}+\ H_2O \qquad (1)$$

In a non-growth medium, if oxygen is the only rate liming substrate, the kinetics of oxidation can be described by the following Michaelis–Menten equation: The steady state mass balance equation for oxygen in the nanoporous latex cellcoat can be written as follows:

$$r_p = \frac{k_{cat} c_0 X_{cat}}{K_0 + c_0} \qquad (2)$$

where r_p is the L-sorbose formation rate, k_{cat} is the rate constant for L-sorbose, c_0 and X_{cat} are the dissolved oxygen and active G. oxydans concentration of oxygen respectively and K_0 is the saturation constant for oxygen. The mass balance equation for oxygen in cellcoat is

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$$D_{eff.cc}\frac{d^2c_0}{dx^2}=\frac{1}{Y_{p/0}}\frac{k_{cat}c_0X_{cat}}{K_0+c_0} \tag{3}$$

where c_0 is the concentration of oxygen, $D_{eff.cc}$ is the effective diffusivity of oxygen , $Y_{p/0}$ is the yield of L-sorbose on oxygen and X_{cat} is the concentration of viable cells in the cellcoat. The equation can be solved subject to the following boundary conditions:

$$\frac{dc_0}{dx}=0 \text{ at } x=0 \tag{4}$$

$$c_0=c_1 \text{ at } x=L_{cc} \tag{5}$$

where L_{cc} is the thickness of the cell coat and c_1 is the dissolved oxygen concentration at the interface of cellcoat and topcoat. An additional condition at the interface between the cellcoat and the topcoat is:

$$c_1=c_{0.c}-L_{TC}\frac{D_{eff.cc}}{D_{eff.Tc}}\left(\frac{dc_0}{dx}\right)_{x=Lcc} \tag{6}$$

The oxygen volumetric consumption rate of the coating can be reported in terms of the effectiveness factor, η_{cc}

$$\eta_{cc}=\frac{(1+c_{0.c}/K_0)L_{cc}}{\phi_{cc}{}^2c_{0.c}}\left(\frac{dc_0}{dx}\right)_{x=Lcc} \tag{7}$$

where ϕ_{cc} is the Thiele modulus:

$$\phi_{cc}=Lcc\sqrt{\frac{k_{cat}X_{cat}}{Y_{P/0}K_0D_{eff.cc}}} \tag{8}$$

2.1 Normalised form

By defining the following dimensionless variables

$$U=\frac{c}{c_1} \text{ and } X=\frac{x}{L_{cc}} \tag{9}$$

The non-linear Eq. (3) becomes in dimensionless form as follows:

$$\frac{d^2U}{dX^2}=\phi_{cc}{}^2\frac{U}{(1+\beta U)} \tag{10}$$

where the dimensionless parameters

$$\phi_{cc}{}^2=\frac{k_{cat}X_{cat}L_{cc}{}^2}{K_0Y_{P/0}D_{eff,cc}} \text{ and } \beta=\frac{c_1}{K_0} \tag{11}$$

The boundary conditions reduces to

$$\frac{dU}{dX}=0 \text{ at } X=0 \tag{12}$$

$$U=1 \text{ at } X=1 \tag{13}$$

The dimensionless effective factor is

$$\eta=\frac{1+\beta}{\phi_{cc}{}^2}\left(\frac{dU}{dX}\right)_{X=1} \tag{14}$$

III. ANALYTICAL SOLUTION OF THE CONCENTRATION USING NEW APPROACH TO HOMOTOPY PERTURBATION METHOD

We use a kind of analytical method called new homotopy perturbation method (NHPM), to solve nonlinear system of second order boundary value problems. The NHPM yields solutions in convergent series form with easily computable terms, and in some cases, yields exact solutions in one iteration. This method can be applied directly to the second order boundary systems needless of converting to the first order initial systems. In the present paper, the system of BVPs will be solved by the NHPM which is introduced by Aminikhah and Hemmatnezhad [7]. NHPM has been successfully applied to stiff systems of ordinary differential equations, initial-type differential equations of heat transfer, nonlinear strongly differential equations, and stiff delay differential equations [8]. Using new approach of homotopy perturbation method (Appendix A), the dimensionless concentration is obtained as follows:

$$U(X)=2(l_1+l_4-l_5)\cosh(sX)+2l_2X\sinh(sX)-l_3(2\cosh(2sX)-6) \tag{15}$$

where

$$s=\sqrt{\frac{\phi_{cc}{}^2}{1+\beta}};\ l_1=\frac{1}{2\cosh(s)};$$
$$l_2=\frac{(\phi_{cc}{}^2-s^2)l_1}{2s};\ l_3=\frac{\beta l_1{}^2}{3};$$
$$l_4=l_1l_3(2\cosh(2s)-6);\ l_5=2l_1l_2\sinh(s) \tag{16}$$

The effectiveness factor is:

$$\eta=\frac{1+\beta}{\phi_{cc}{}^2}\left[2s(l_1+l_4-l_5)\sinh(s)+2l_2(s\cosh(s)+\sinh(s))-4sl_3(\sinh(2s))\right] \tag{17}$$

IV. ANALYTICAL SOLUTION OF THE CONCENTRATION USING ADOMIAN DECOMPOSITION METHOD

In the recent years, much attention is devoted to the application of the Adomian decomposition method to the solution of various scientific models [9-14] which was introduced by Adomian [15,16].Recently Praveen and co-author (2011)obtain the analytical expression of the oxygen concentration using Adomian decomposition method as follows:

$$U(X)=1+\frac{\phi_{cc}{}^2(X^2-1)}{2(1+\beta)}+\frac{\phi_{cc}{}^4(X^4-6X^2+5)}{24(1+\beta)^3} \tag{18}$$

The above equation is valid provided

$$1-\frac{\phi_{cc}{}^2}{2(1+\beta)}+\frac{5\phi_{cc}{}^4}{24(1+\beta)^3}\geq o \tag{19}$$

V. RESULTS AND DISCUSSION

The kinetic response of a microchannel bioreactor depends on the concentrations of oxygen. The concentrations of oxygen depends on the two

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parameters ϕ_{cc}^2 and β. Thiele modulus ϕ^2, represents the ratio of the characteristic time of the enzymatic reaction to that of substrate diffusion. Small values of the Thiele modulus indicate surface reaction controls and a significant amount of the reactant diffuses well into the pellet interior without reacting. Large values of the Thiele modulus indicate that the surface reaction is rapid and that the reactant is consumed very close to the external pellet surface and very little penetrates into the interior of the pellet.

Praveen and the co-author [6] obtained the analytical expression for the dimensionless concentration for the non-steady state condition using Adomin decomposition method. In this work the non-steady state approximate analytical expressions for the concentration of oxygen is given by Eq.(15) and Figures 1(a)-(b) and 2(a)-(d) represent the normalized steady state substrate dissolved oxygen concentration versus distance. The concentration of substrates were calculated for various values of saturation parameter values β and reaction diffusion parameter ϕ_{cc}^2 respectively using the (Eq. 15).

Figure 1. shows the concentration profile for three different values of the Thiele modulus. Small values of the Thiele modulus indicates that the concentration variation inside the particle is negligible and for a significant amount of the thiele modulus the concentration profile is flat. Large values of the Thiele modulus indicate that the concentration profile very close to the external surface.

From the Figures 1(a)-(b), it is evident that when reaction diffusion parameter ϕ_{cc}^2 decreases concentration increases. From the figure 2(a)-(d), it is observed that the oxygen concentration increases when the saturation parameter b decreases. We can conclude that the results gives satisfactory agreement with simulation results and Eq. (18) for all possible values of β and ϕ_{cc}^2.

From these figures, it is inferred that the concentration increases slowly when the distance increases. From Figure 2(a), it is observed that, when β decreases the concentration also increases for all values of kinetic parameter β and for fixed value of ϕ_{cc}^2.The analytical result (Eq.(15)) is compared with the previous work Eq.(18) and it is observed that the satisfactory agreement is noted.

From Figures 2(a)-(d), It is observed that when β decreases the concentration increases for all values of other parameters. The error of deviation for the Eq. (15) and Eq.(18) with the numerical result is given in the tables (1-4) . From the table it is inferred that, the NHPM gives better approximation when β is less than 1and for all values of the parameter ϕ_{cc}^2 and ADM gives less error when β is greater than 1 and $\phi_{cc}^2 > 5$.

The variation in effectiveness factor η, obtained for various values of β is shown in Figure 3. From this figure, it is observed that the oxygen volumetric consumption rate of the coating decreases when the saturation parameter β decreases.

All the above results are also confirmed infigures 4 and 5. Sensitivity analysis of the parameters is given in figure 6. From this figure ,it is inferred that the reaction diffusion parameter has more impact than the saturation parameter. The concentration of the substrate is minimum at $X = 0$ and the minimum value is

$$U = 1 - \frac{\phi_{cc}^2}{2(1+\beta)} + \frac{5\phi_{cc}^4}{24(1+\beta)^3}.$$

VI. CONCLUSION

A theoretical model describing the process of reaction and diffusion of steady state elemental balance for oxygen concentration in the nano-porous latex cellcoat is presented. We have derived the transport and kinetics in terms of the reaction/diffusion parameter ϕ_{cc}^2 and the saturation parameter β .In this paper, the NHPM had been successfully applied to find the solution of a non-linear ordinary differential equation. All The result of the present method are in excellent agreement with those obtained by the ADM. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

Our analytical result NHPM coincide with the numerical result when β is less than 1 and for all values of the parameter ϕ_{cc}^2 wheras the previous analytical result (ADM) (Eq.(18))gives good agreement with the numerical result when β is greater than 1 and ϕ_{cc}^2 approximately greater than 5. Moreover, we have also presented an analytical expression for the steady-state effectiveness factor.

APPENDIX A. **Approximate analytical solution of the nonlinear Eq. (10) using a new approach to the homotopy perturbation method .**

In this appendix, we have indicated how to determine the solution of Eq. (10) using the boundary condition Eqs. (12) and (13).In order to solve Eq. (10), we first construct the homotopy for the equation as follows:

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$$(1-p)\left(\frac{d^2U}{dX^2}-\frac{\phi_{cc}{}^2U}{1+\beta U(X=1)}\right)+ p\left((1+\beta U)\frac{d^2U}{dX^2}-\phi_{cc}{}^2U\right)=0 \quad (A.1)$$

$$(1-p)\left(\frac{d^2U}{dX^2}-\frac{\phi_{cc}{}^2U}{1+\beta}\right)+p\left((1+\beta U)\frac{d^2U}{dX^2}-\phi_{cc}{}^2U\right)=0 \quad (A.2)$$

The approximate solution of Eq. (10) is

$$U=U_0+U_1p+U_2p^2+.... \quad (A.3)$$

Substituting equation (A.3) into equation (A.2) and arranging the co-efficient of powers p, we obtain

$$p^0:\frac{d^2U_0}{dX^2}-\frac{\phi_{cc}{}^2U_0}{1+\beta}=0 \quad (A.4)$$

$$p^1:\frac{d^2U_1}{dX^2}-\frac{\phi_{cc}{}^2U_1}{1+\beta}+(1+\beta)\frac{d^2U_0}{dX^2}-\phi_{cc}{}^2U_0=0 \quad (A.5)$$

The boundary conditions in Eqs. (A.4) and (A.5) becomes

At $X=0$, $\frac{dU_0}{dX}=0$ and $\frac{dU_i}{dX}=0\ \forall i=1,2,3...$ (A.6)

$X=1, U_0=1$ and $U_i=0\ \forall i=1,2,3...$ (A.7)

Solving the Eqs. (A.4) and (A.5), and using the boundary conditions and (A.6) and
(A.7) we can obtain the following results

$$U_0(X)=l_1(2\cosh(sX)) \text{ where } l_1=\frac{1}{2\cosh s} \quad (A.8)$$

$$U_1(X)=(l_4-l_5)(2\cosh\ (sX))+2\,l_2X\sinh\ (sX)-l_3(2\cosh\ (2sX)-6) \quad (A.9)$$

$$s=\sqrt{\frac{\phi_{cc}{}^2}{1+\beta}};\quad l_1=\frac{1}{2\cosh(s)};\ l_2=\frac{(\phi_{cc}{}^2-s^2)\,l_1}{2s};\ l_3=\frac{\beta l_1{}^2}{3};$$

$$l_4=l_1l_3(2\cosh(2s)-6);\ l_5=2l_1l_2(\sinh(s))$$

Adding (A. 9) to (A. 10) we get Eq. (15) in the text.

Nomenclature

D_{eff}	Oxygen effective diffusivity	$(m^2 s^{-1})$
c	Dissolved oxygen concentration	$(g\,l^{-1})$
K_{cat}	Reaction rate constant	$(mol\ CFU^{-1}\,s^{-1})$
X_{cat}	G.oxydans cell density in the cellcoat	$(CFU\,m^{-3})$
K_o	Michaelis-Menten constant for oxygen	$(m\,g\,l^{-1})$
$Y_{P/0}$	Yield coefficient of L-sorbose on oxygen	None
L_{cc}	Cellcoat thickness	(m)
η	Effectiveness factor	None
ϕ	Thiele modulus	None
β	Oxygen concentration in the coating	None

APPENDIX B. Scilab program for the numerical solution of Eq. (10)

```
function pdex1
m = 0;
x = linspace (0, 1);
t = linspace (0, 10000);
sol = pdepe (m, @pdex1pde, @pdex1ic, @pdex1bc, x, t);
% Extract the first solution component as u.
u = sol (, 1);
figure
plot(x, u (end, :))
xlabel (Distance x)
ylabel (u(x, 2))
% ---------------------------------------------------------------
function [c, f, s] = pdex1pde(x, t, u, DuDx)
c = 1;
f = DuDx;
r=;a=;
s =-(r*u)/ ((1+ (u*a)));
% ---------------------------------------------------------------
function u0 = pdex1ic(x)
u0 = 0;
% ---------------------------------------------------------------
function [pl,ql,pr,qr] = pdex1bc(xl,ul,xr,ur,t)
pl = 0;
ql = 1;
pr = ur-1;
qr = 0;
```

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L_{TC}	Topcoat thickness	(m)
U	Oxygen concentration	None
X	Distance	None

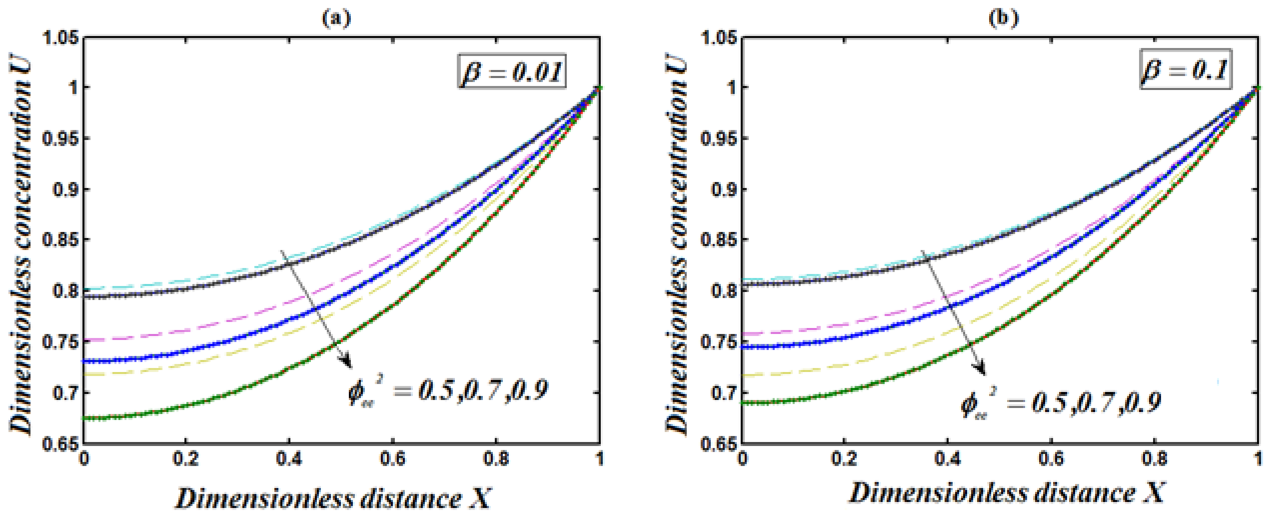

Fig. 1(a)–(b): Normalised concentration profile $U(X)$ *as a function of dimensionless parameter* $X = x/L_{cc}$ *. The concentrations were computed using Eq. (15) and (18) for various values of the* $\phi_{cc}{}^2$ *and for the values (a)* $\beta = 0.1$ *, (b)* $\beta = 0.01$ *. (___) denotes Eq. (15), (_ _ _) denotes Eq. (18) and (….) denotes the numerical simulation.*

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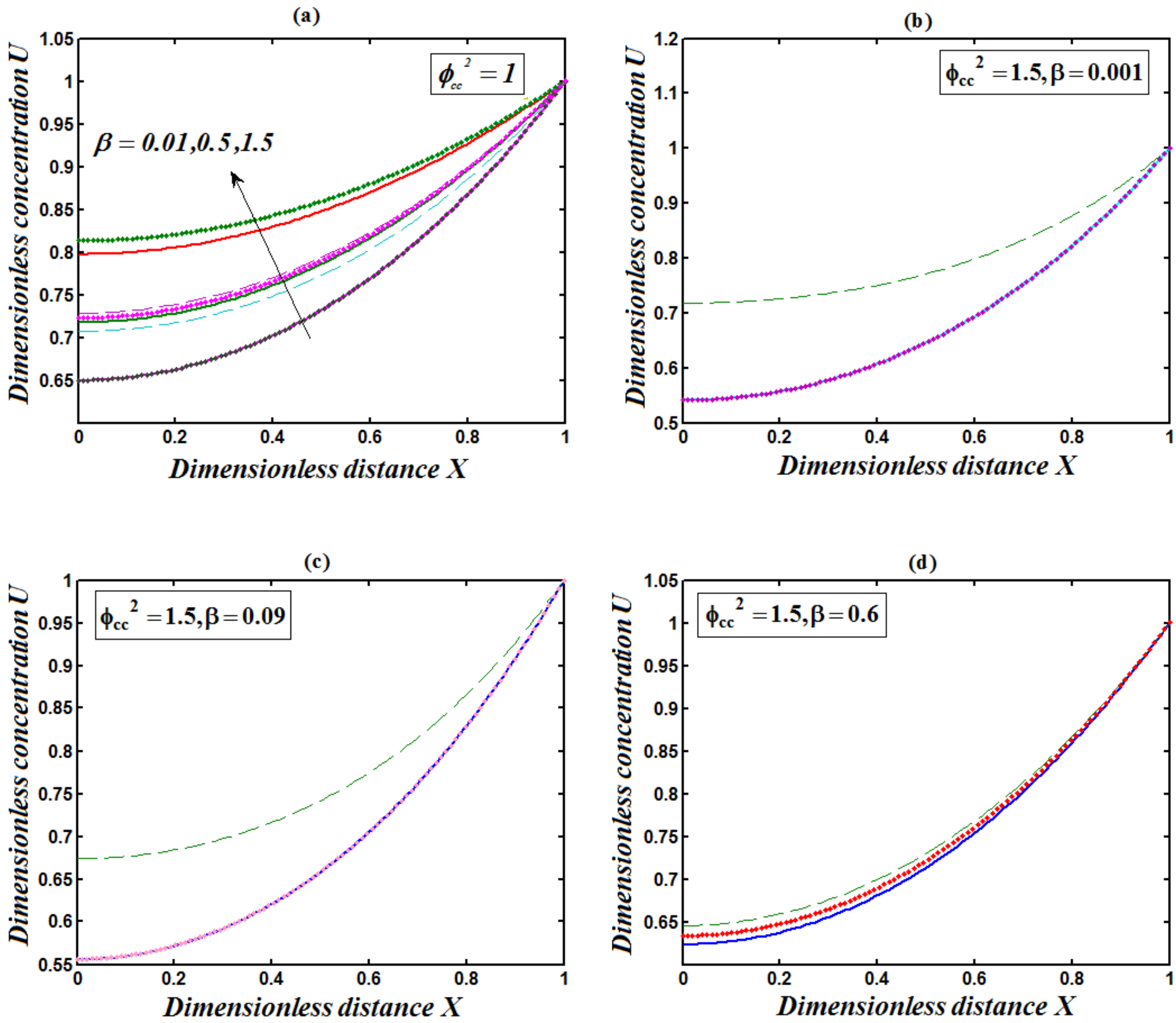

Fig. 2(a)–(d): Normalised concentration profile $U(X)$ as a function of dimensionless parameter $X = x/L_{cc}$. The concentrations were computed using Eqs. (15) and (18) for various values of the β and for the values (a) $\phi_{cc}^{2} = 1$, (b-d) $\phi_{cc}^{2} = 1.5$. (____) denotes Eq. (15), (_ _ _) denotes Eq.(18) and (....) denotes the numerical simulation.

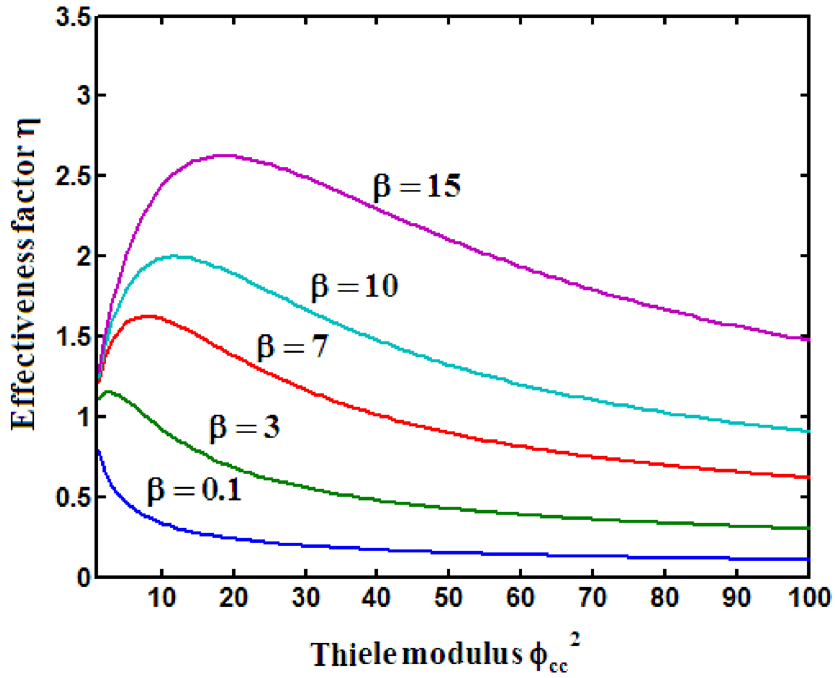

Fig.3: The effectiveness factor η *as a function of Thiele modulus* $\phi_{cc} = L_{cc}\sqrt{k_{cat}X_{cat}/K_0 Y_{P/0} D_{eff.cc}}$ *.The effectiveness factor were computed using Eq. (16) various value of* β *the by fixing* $\phi_{cc}^2 = 0-100$

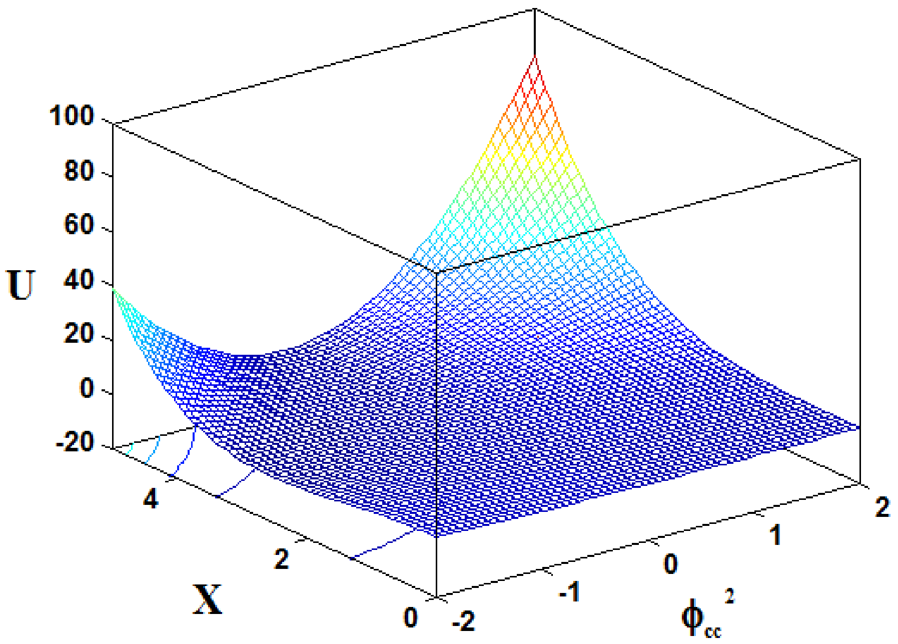

Fig. 4: Three dimension plot for oxygen concentration using the approximate analytical solution Eq.(15) for the fixed value $\beta = 0.1$ *and* ϕ_{cc}^2 *varies from -2 to 2 and* X *varies from 0-5*

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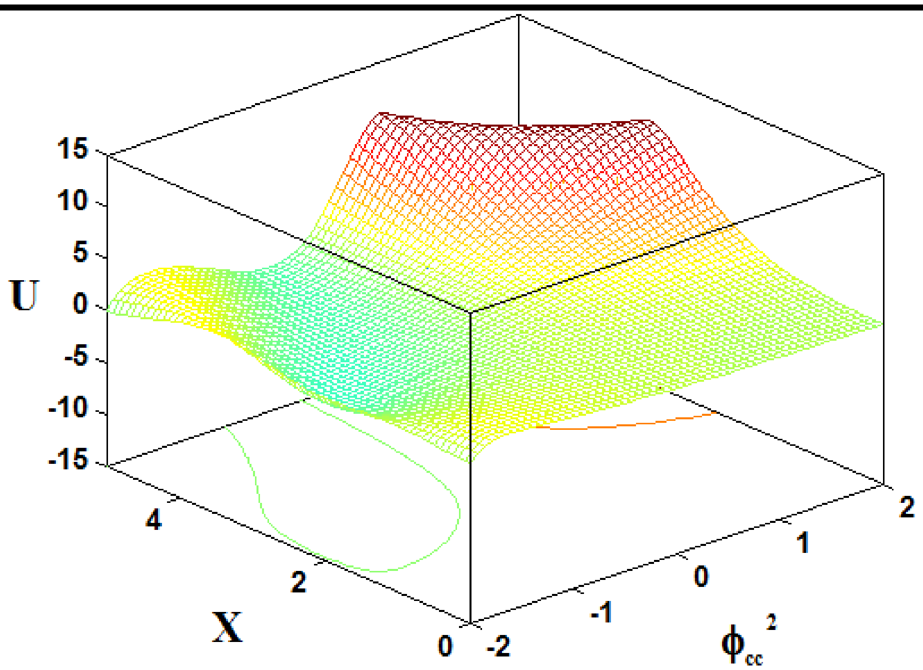

Fig. 5: Three dimension plot for oxygen concentration using the approximate analytical solutions Eq. (18) for the fixed value $\beta = 0.1$ *and* ϕ_{cc}^{2} *varies from -2 to 2 and* X *varies from 0-5*

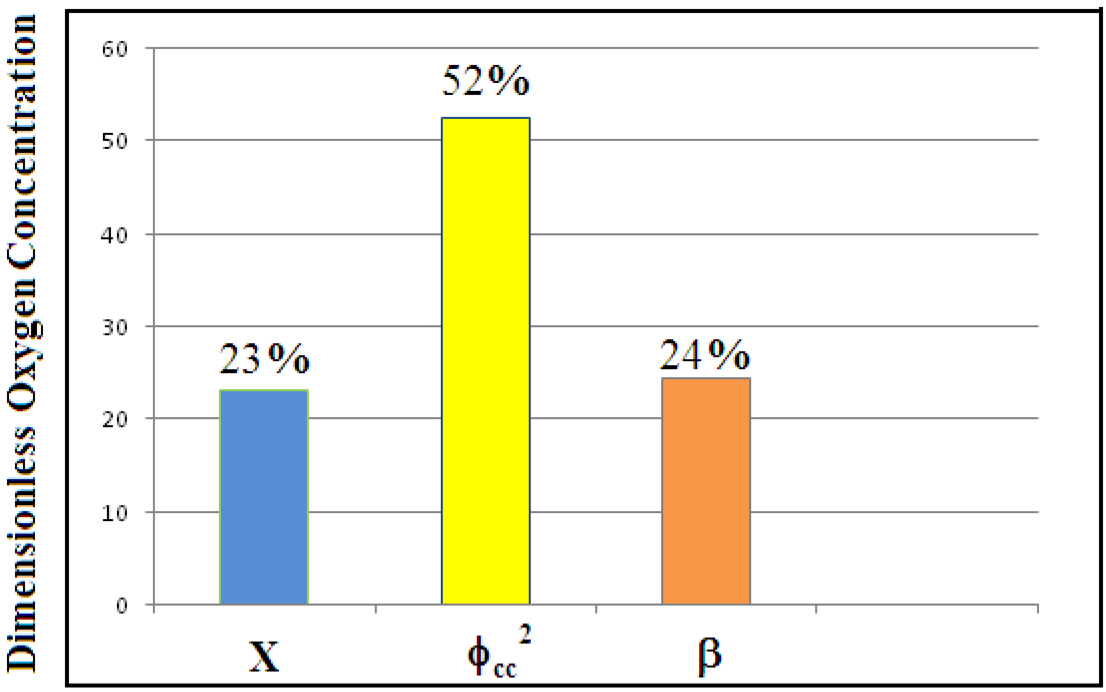

Fig. 6: Sensitivity of parameters : Percentage change in dimensionless oxygen concentration when $X = 4$, $\phi_{cc}^{2} = 0.55$, $\beta = 2$

Table 1: *Comparison of our analytical result (Eq. (15)) of $U(X)$ with the previous result (Eq. (18)) for various values of $\phi_{CC}{}^2$ and for the fixed value of $\beta = 0.01$.*

X	$\phi_{cc}{}^2 = 0.5$					$\phi_{cc}{}^2 = 0.9$					$\phi_{cc}{}^2 = 5$					$\phi_{cc}{}^2 = 10$				
	Numerical	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**
0	0.795	0.803	0.795	1.050	0.000	0.675	0.718	0.675	6.373	0.000	0.212	3.580	0.212	1587.761	0.006	0.085	16.270	0.085	19089.646	0.016
0.2	0.803	0.811	0.803	0.989	0.000	0.687	0.728	0.687	5.956	0.000	0.234	3.438	0.234	1371.497	0.006	0.102	15.504	0.102	15055.000	0.015
0.4	0.826	0.833	0.826	0.818	0.000	0.724	0.759	0.724	4.813	0.000	0.303	3.031	0.303	901.883	0.005	0.162	13.283	0.162	8095.374	0.013
0.6	0.867	0.872	0.867	0.567	0.000	0.787	0.812	0.787	3.221	0.000	0.433	2.418	0.433	458.716	0.003	0.289	9.841	0.289	3307.836	0.009
0.8	0.924	0.927	0.924	0.280	0.000	0.878	0.891	0.878	1.519	0.000	0.651	1.696	0.651	160.618	0.002	0.535	5.565	0.535	941.209	0.005
1.0	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000
	Average % of error			0.617	0.000	Average % of error			3.647	0.000	Average % of error			746.746	0.004	Average % of error			7748.177	0.010

Table 2: Comparison of our analytical result (Eq. (15)) of $U(X)$ with the previous result (Eq. (18)) for various values of $\phi_{CC}{}^2$ and for the fixed value of $\beta=0.1$

X	$\phi_{cc}{}^2=0.5$					$\phi_{cc}{}^2=0.9$					$\phi_{cc}{}^2=5$					$\phi_{cc}{}^2=10$				
	Numerical	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**
0	0.807	0.812	0.807	0.642	0.022	0.690	0.718	0.690	3.994	0.047	0.219	2.640	0.218	1103.999	0.396	0.088	12.107	0.087	13732.705	0.960
0.2	0.814	0.819	0.814	0.605	0.021	0.702	0.728	0.701	3.738	0.044	0.241	2.545	0.240	955.434	0.349	0.105	11.542	0.105	10845.324	0.787
0.4	0.837	0.841	0.837	0.502	0.017	0.737	0.759	0.737	3.033	0.036	0.311	2.273	0.310	631.382	0.245	0.167	9.909	0.166	5851.178	0.473
0.6	0.875	0.878	0.875	0.350	0.012	0.797	0.813	0.797	2.043	0.025	0.442	1.870	0.441	323.368	0.139	0.295	7.387	0.294	2402.737	0.231
0.8	0.929	0.930	0.929	0.173	0.007	0.884	0.893	0.884	0.971	0.012	0.658	1.410	0.658	114.246	0.056	0.542	4.277	0.542	689.211	0.083
1.0	1.000	1.000	1.000	0.001	0.001	1.000	1.000	1.000	0.001	0.001	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.001	0.001
	Average % of error			0.379	0.013	Average % of error			2.297	0.028	Average % of error			521.405	0.197	Average % of error			5586.859	0.422

Table 3: *Comparison of our analytical result (Eq. (15)) of $U(X)$ with the previous result (Eq. (18)) for various values of ϕ_{CC}^{2} and for the fixed value of $\beta=1$*

X	$\phi_{cc}^{2}=0.5$					$\phi_{cc}^{2}=0.9$					$\phi_{cc}^{2}=5$					$\phi_{cc}^{2}=10$				
	Numerical	Previous work Eq.(18)	This work Eq.(15)	% of deviation Eq.(18)	% of deviation Eq.(15)	Numerical	Previous work Eq.(18)	This work Eq.(15)	% of deviation Eq.(18)	% of deviation Eq.(15)	Numerical	Previous work Eq.(18)	This work Eq.(15)	% of deviation Eq.(18)	% of deviation Eq.(15)	Numerical	Previous work Eq.(18)	This work Eq.(15)	% of deviation Eq.(18)	% of deviation Eq.(15)
0.000	0.882	0.882	0.877	0.001	0.494	0.796	0.796	0.786	0.003	1.319	0.297	0.401	0.251	34.981	15.671	0.118	1.104	0.059	838.598	49.478
0.200	0.886	0.886	0.882	0.001	0.469	0.804	0.804	0.794	0.004	1.247	0.320	0.420	0.275	31.150	14.046	0.139	1.080	0.083	675.539	40.666
0.400	0.900	0.900	0.897	0.000	0.396	0.828	0.828	0.819	0.006	1.042	0.392	0.479	0.352	22.189	10.219	0.211	1.018	0.159	382.356	24.485
0.600	0.924	0.924	0.921	0.000	0.283	0.868	0.869	0.862	0.007	0.735	0.521	0.587	0.490	12.577	6.017	0.353	0.947	0.311	167.800	11.946
0.800	0.957	0.957	0.956	0.000	0.146	0.926	0.926	0.922	0.005	0.369	0.719	0.754	0.701	4.965	2.497	0.601	0.918	0.576	52.545	4.295
1.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000
	Average % of error			0.000	0.298	Average % of error			0.004	0.785	Average % of error			17.644	8.075	Average % of error			352.806	21.812

***Table 4**: Comparison of our analytical result (Eq. (15)) of* $U(X)$ *with the previous result (Eq. (18)) for various values of* $\phi_{CC}{}^2$ *and for the fixed value of* $\beta = 5$

X	$\phi_{cc}{}^2 = 0.5$					$\phi_{cc}{}^2 = 0.9$					$\phi_{cc}{}^2 = 5$					$\phi_{cc}{}^2 = 10$				
	Numerical	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**	**Numerical**	**Previous work Eq.(18)**	**This work Eq.(15)**	**% of deviation Eq.(18)**	**% of deviation Eq.(15)**
0.000	0.959	0.959	0.953	0.001	0.561	0.926	0.926	0.910	0.004	1.720	0.615	0.607	0.399	1.169	35.122	0.326	0.263	-0.065	19.361	119.792
0.200	0.960	0.960	0.955	0.001	0.533	0.929	0.929	0.914	0.004	1.634	0.630	0.623	0.423	1.075	32.798	0.351	0.292	-0.025	16.900	107.084
0.400	0.965	0.965	0.961	0.000	0.453	0.938	0.938	0.925	0.003	1.385	0.675	0.670	0.497	0.832	26.466	0.427	0.378	0.097	11.434	77.404
0.600	0.973	0.973	0.970	0.000	0.329	0.952	0.952	0.943	0.002	0.999	0.752	0.748	0.619	0.524	17.703	0.558	0.524	0.306	6.088	45.230
0.800	0.985	0.985	0.983	0.000	0.172	0.973	0.973	0.968	0.001	0.518	0.860	0.858	0.788	0.233	8.345	0.747	0.730	0.608	2.295	18.689
1.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000	1.000	1.000	1.000	0.000	0.000
	Average % of error			0.000	0.341	Average % of error			0.003	1.043	Average % of error			0.639	20.072	Average % of error			9.346	61.367

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